Unit 2: Limits and Continuity

Name: ____________________

The **limit** of a function is ____

____.

The graph MUST approach the same y-value from both sides (directions) for the limit to exist!

NOTATION for the Limit Form:

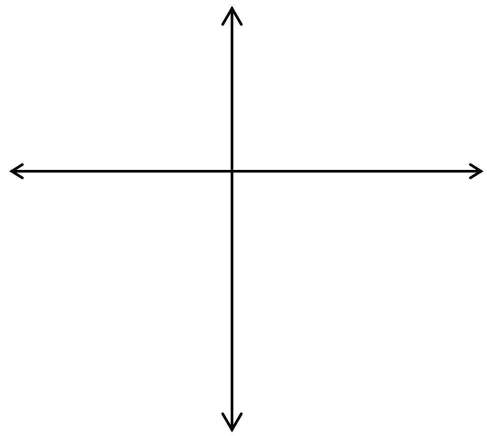
Examples:

1. What y-value does the graph of $y = x^2 + 2$ get closer to as x approaches 2?

OR $\lim_{x \to 2} (x^2 + 2) = ?$

x	y
1	3
1.5	4.25
1.9	5.61
1.99	5.96
1.999	5.996

x	y
3	11
2.5	8.25
2.1	6.41
2.01	6.04
2.001	6.004



$\therefore$

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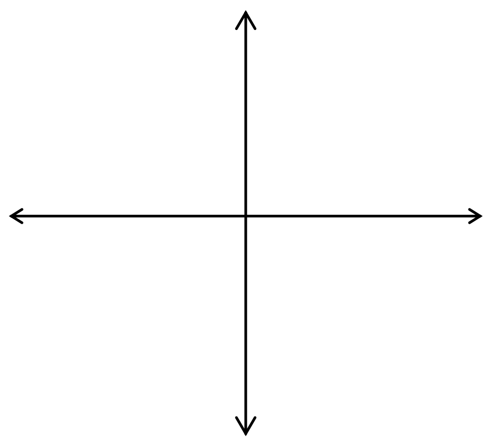
2. Find $\lim_{x \to 1} \left(\frac{x^2 - 3x + 2}{x - 1} \right) = ?$

X	y
0	-2
0.5	-1.5
0.9	-1.1
0.99	-1.01
0.999	-1.001

X	Y
2	0
1.5	-0.5
1.1	-0.9
1.01	-0.99
1.001	-0.999



$y = \frac{x^2 - 3x + 2}{x - 1} =$

$\therefore$

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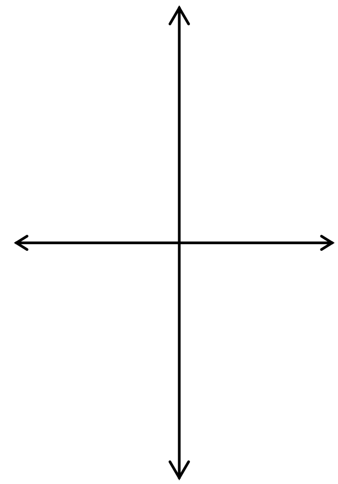
3. Find $\lim\limits_{x \to 1}\left(\dfrac{x^2 - 2x - 1}{x - 1}\right) \Rightarrow$

X	y
0	1
0.5	3.5
0.9	19.9
0.99	199.99
0.999	1999.99

X	Y
2	-1
1.5	-3.5
1.1	-19.9
1.01	-199.99
1.001	-1999.99



$$y = \frac{x^2 - 2x - 1}{x - 1} =$$

$\therefore$

(NOTE): ____

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One-Sided Limits

Left-sided

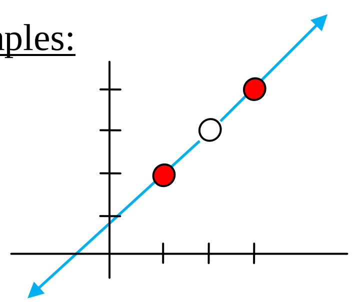
$$\boxed{\lim_{x \to c^-} f(x) = L}$$

Right-sided

$$\boxed{\lim_{x \to c^+} f(x) = L}$$

Examples:

1.

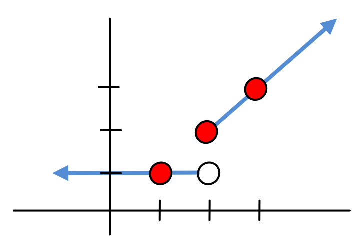


$\lim\limits_{x \to 2^-} f(x) =$ ____

$\lim\limits_{x \to 2^+} f(x) =$ ____

$\therefore \lim\limits_{x \to 2} f(x) =$ ____

2.



$\lim\limits_{x \to 2^-} f(x) =$ ____

$\lim\limits_{x \to 2^+} f(x) =$ ____

$\therefore \lim\limits_{x \to 2} f(x) =$ ____

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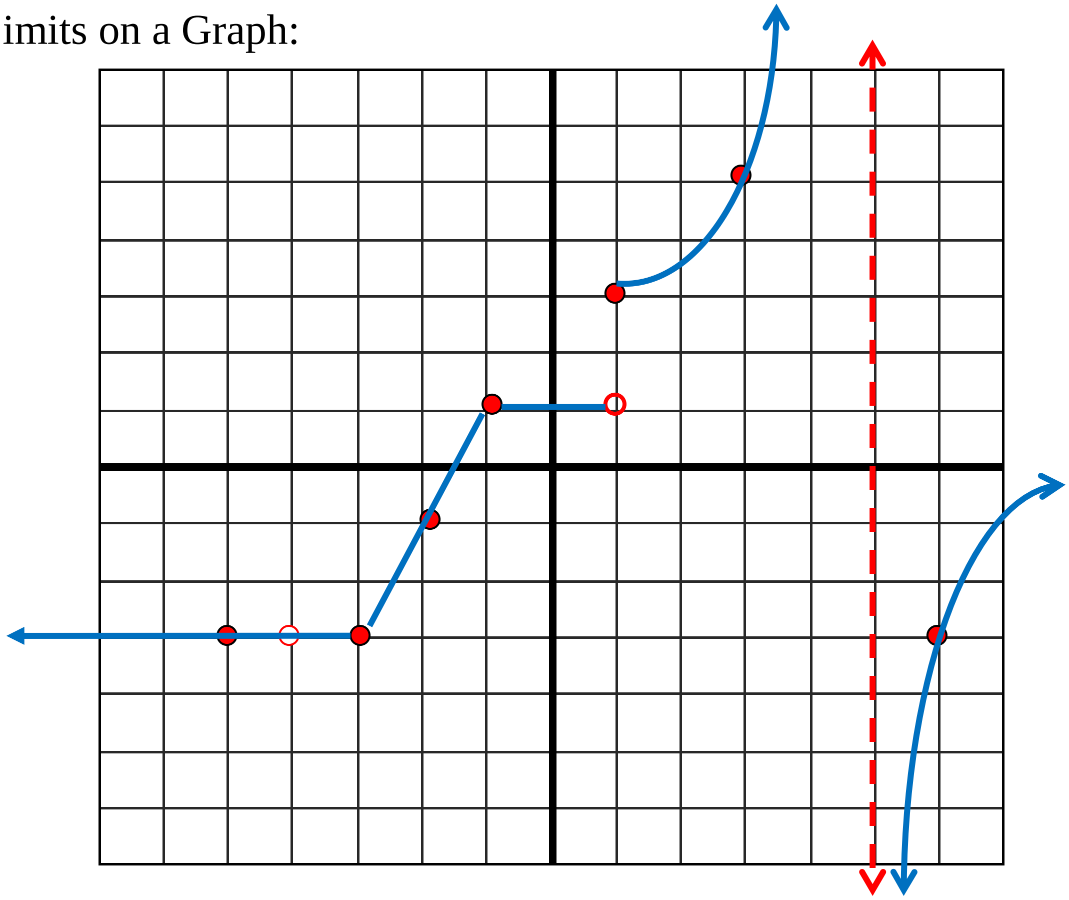
Limits on a Graph:



$\lim_{x\to 1^-} f(x) =$ $\lim_{x\to 1^+} f(x) =$ $\lim_{x\to 1} f(x) =$ f (1) =

$\lim_{x\to -2^-} f(x) =$ $\lim_{x\to -2^+} f(x) =$ $\lim_{x\to -2} f(x) =$ f (-2) =

$\lim_{x\to 5^-} f(x) =$ $\lim_{x\to 5^+} f(x) =$ $\lim_{x\to 5} f(x) =$ f (5) =

$\lim_{x\to 3^-} f(x) =$ $\lim_{x\to 3^+} f(x) =$ $\lim_{x\to 3} f(x) =$ f (3) =

$\lim_{x\to -4^-} f(x) =$ $\lim_{x\to -4^+} f(x) =$ $\lim_{x\to -4} f(x) =$ f (-4) =

Name all values of x for which the function does not have a limit: ___________

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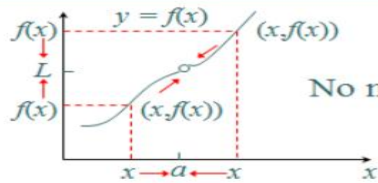
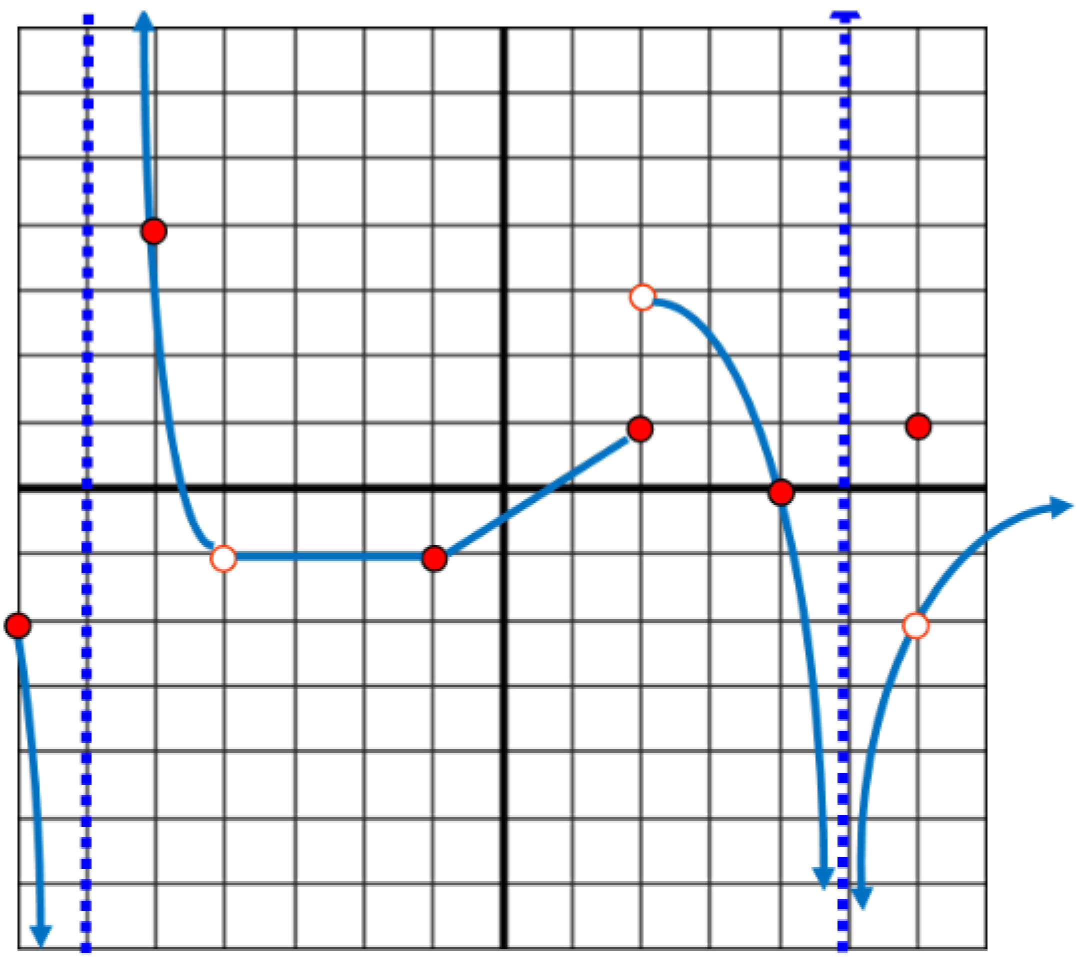
Definition of Limit of a Function

Suppose that the function $f(x)$ is defined for all values of x near a, but not necessarily at a. If as x approaches a (without actually attaining the value a), $f(x)$ approaches the number L, then we say that L is the limit of $f(x)$ as x approaches a, and write

$$\lim_{x \to a} f(x) = L$$



No matter how x approaches a, $f(x)$ approaches L.



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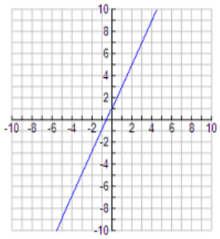
Examples:

1) 

$f(x) = 2x + 1$

$f(2) =$

2) $f(x) = \dfrac{x^2 - 1}{x - 1}$

$f(1) =$

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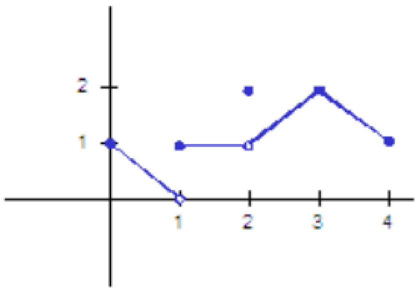
3)



a) $\lim\limits_{x \to 1^+} f(x) =$

$\lim\limits_{x \to 1^-} f(x) =$

$f(1) =$

b) $\lim\limits_{x \to 2^+} f(x) =$

$\lim\limits_{x \to 2^-} f(x) =$

$f(2) =$

c) $\lim\limits_{x \to 3^+} f(x) =$

$\lim\limits_{x \to 3^-} f(x) =$

$f(3) =$

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4) $f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, \; if & x \neq 2 \\ 7 \quad \text{if} & x = 2 \end{cases}$

5) $f(x) = \begin{cases} 3 + x^2, \; if & x < -2 \\ 9 - x^2, \; if & x \geq -2 \end{cases}$

$f(2) =$

$\lim\limits_{x \to 2^+} f(x) =$

$\lim\limits_{x \to 2^-} f(x) =$

$f(2) =$

$\lim\limits_{x \to 2^+} f(x) =$

$\lim\limits_{x \to 2^-} f(x) =$